

Contents

Preface	xiii
About the Author	xv
Dependency Chart	xvii
Acknowledgments	xix
1 An Overview of the Subject	1
Basic Concepts	1
Functions	4
One-to-One and Onto Functions, Bijections	5
Inverse Functions	7
Substitution Ciphers	8
Attacks on Cryptosystems	12
The Vigenère Cipher	15
The Playfair Cipher	18
The One-Time Pad, Perfect Secrecy	25
Chapter 1 Exercises	28
Chapter 1 Computer Implementations and Exercises	35
Vector/String Conversions	35
Integer/Text Conversions	36
Programming Basic Ciphers with Integer Arithmetic	38
Computer-Generated Random Numbers	39
2 Divisibility and Modular Arithmetic	43
Divisibility	43
Primes	44
Greatest Common Divisors and Relatively Prime Integers	46
The Division Algorithm	47
The Euclidean Algorithm	48
Modular Arithmetic and Congruences	52
Modular Integer Systems	58
Modular Inverses	60
Extended Euclidean Algorithm	61
Solving Linear Congruences	64
Summary of Procedure for Solving the Single Linear Congruence (Equation 2.2)	66
The Chinese Remainder Theorem	67
Chapter 2 Exercises	71
Chapter 2 Computer Implementations and Exercises	85

3	The Evolution of Codemaking until the Computer Era.	91
	Ancient Codes	91
	Formal Definition of a Cryptosystem	94
	Affine Ciphers	96
	Steganography	100
	Nulls	102
	Homophones	105
	Composition of Functions	109
	Tabular Form Notation for Permutations	110
	The Enigma Machines	111
	Cycles (Cyclic Permutations)	114
	Dissection of the Enigma Machine into Permutations	119
	Special Properties of All Enigma Machines	126
	Chapter 3 Exercises	127
	Chapter 3 Computer Implementations and Exercises	136
	Computer Representations of Permutations	140
4	Matrices and the Hill Cryptosystem	145
	The Anatomy of a Matrix	145
	Matrix Addition, Subtraction, and Scalar Multiplication	146
	Matrix Multiplication	147
	Preview of the Fact That Matrix Multiplication Is Associative	149
	Matrix Arithmetic	149
	Definition of an Invertible (Square) Matrix	151
	The Determinant of a Square Matrix	153
	Inverses of 2×2 Matrices	155
	The Transpose of a Matrix	156
	Modular Integer Matrices	156
	The Classical Adjoint (for Matrix Inversions)	159
	The Hill Cryptosystem	162
	Chapter 4 Exercises	166
	Chapter 4 Computer Implementations and Exercises	174
5	The Evolution of Codebreaking until the Computer Era.	181
	Frequency Analysis Attacks	181
	The Demise of the Vigenère Cipher	187
	The Babbage/Kasiski Attack	188
	The Friedman Attack	192
	The Index of Coincidence	193
	Expected Values of the Index of Coincidence	193
	How Enigmas Were Attacked	201
	German Usage Protocols for Enigmas	202
	The Polish Codebreakers	203
	Rejewski's Attack	203
	Invariance of Cycle Decomposition Form	205
	Alan Turing and Bletchley Park	206
	Chapter 5 Exercises	208
	Chapter 5 Computer Implementations and Exercises	214

Programs to Aid in Frequency Analysis 214
 Programs to Aid in the Babbage/Kasiski Attack 215
 Programs Related to the Friedman Attack 218

6 Representation and Arithmetic of Integers in Different Bases 221

Representation of Integers in Different Bases 221
 Hex(adecimal) and Binary Expansions 224
 Addition Algorithm with Base b Expansions 229
 Subtraction Algorithm with Base b Expansions 231
 Multiplication Algorithm in Base b Expansions 234
 Arithmetic with Large Integers 237
 Fast Modular Exponentiation 239
 Chapter 6 Exercises 241
 Chapter 6 Computer Implementations and Exercises 248

7 Block Cryptosystems and the Data Encryption Standard (DES) . . . 251

The Evolution of Computers into Cryptosystems 251
 DES Is Adopted to Fulfill an Important Need 252
 The XOR Operation 254
 Feistel Cryptosystems 255
 A Scaled-Down Version of DES 258
 DES 265
 The Fall of DES 272
 Triple DES 273
 Modes of Operation for Block Cryptosystems 274
 Electronic Codebook (ECB) Mode 274
 Cipherblock Chaining (CBC) Mode 275
 Cipher Feedback (CFB) Mode 276
 Output Feedback (OFB) Mode 278
 Chapter 7 Exercises 279
 Chapter 7 Computer Implementations and Exercises 286

8 Some Number Theory and Algorithms 293

The Prime Number Theorem 293
 Fermat's Little Theorem 295
 The Euler Phi Function 298
 Euler's Theorem 300
 Modular Orders of Invertible Modular Integers 301
 Primitive Roots 302
 Existence of Primitive Roots 304
 Determination of Primitive Roots 304
 Order of Powers Formula 305
 Prime Number Generation 308
 Fermat's Primality Test 309
 Carmichael Numbers 311
 The Miller–Rabin Test 312
 The Miller–Rabin Test with a Factoring Enhancement 315

The Pollard $p - 1$ Factoring Algorithm 316
 Chapter 8 Exercises 319
 Chapter 8 Computer Implementations and Exercises 325

9 Public Key Cryptography 331

An Informal Analogy for a Public Key Cryptosystem 331
 The Quest for Secure Electronic Key Exchange 332
 One-Way Functions 333
 Review of the Discrete Logarithm Problem 334
 The Diffie–Hellman Key Exchange 336
 The Quest for a Complete Public Key Cryptosystem 337
 The RSA Cryptosystem 338
 Digital Signatures and Authentication 343
 The ElGamal Cryptosystem 345
 Digital Signatures with ElGamal 347
 Knapsack Problems 349
 The Merkle–Hellman Knapsack Cryptosystem 352
 Government Controls on Cryptography 356
 A Security Guarantee for RSA 357
 Chapter 9 Exercises 360
 Chapter 9 Computer Implementations and Exercises 369

10 Finite Fields in General, and $GF(2^8)$ in Particular. 377

Binary Operations 377
 Rings 378
 Fields 381
 $\mathbb{Z}_p[X]$ = the Polynomials with Coefficients in \mathbb{Z}_p 385
 Addition and Multiplication of Polynomials in $\mathbb{Z}_p[X]$ 386
 Vector Representation of Polynomials 387
 $\mathbb{Z}_p[X]$ Is a Ring 388
 Divisibility in $\mathbb{Z}_p[X]$ 389
 The Division Algorithm for $\mathbb{Z}_p[X]$ 391
 Congruences in $\mathbb{Z}_p[X]$ Modulo a Fixed Polynomial 395
 Building Finite Fields from $\mathbb{Z}_p[X]$ 396
 The Fields $GF(2^4)$ and $GF(2^8)$ 399
 The Euclidean Algorithm for Polynomials 404
 Chapter 10 Exercises 406
 Chapter 10 Computer Implementations and Exercises 411

11 The Advanced Encryption Standard (AES) Protocol 417

An Open Call for a Replacement to DES 417
 Nibbles 419
 A Scaled-Down Version of AES 421
 Decryption in the Scaled-Down Version of AES 429
 AES 432
 Byte Representation and Arithmetic 432
 The AES Encryption Algorithm 435

The AES Decryption Algorithm 439
 Security of the AES 440
 Chapter 11 Exercises 441
 Chapter 11 Computer Implementations and Exercises 445

12 Elliptic Curve Cryptography 451
 Elliptic Curves over the Real Numbers 452
 The Addition Operation for Elliptic Curves 454
 Groups 458
 Elliptic Curves over \mathbb{Z}_p 460
 The Variety of Sizes of Modular Elliptic Curves 462
 The Addition Operation for Elliptic Curves over \mathbb{Z}_p 463
 The Discrete Logarithm Problem on Modular Elliptic Curves 466
 An Elliptic Curve Version of the Diffie–Hellman Key Exchange 467
 Fast Integer Multiplication of Points on Modular Elliptic Curves 470
 Representing Plaintexts on Modular Elliptic Curves 471
 An Elliptic Curve Version of the ElGamal Cryptosystem 473
 A Factoring Algorithm Based on Elliptic Curves 475
 Chapter 12 Exercises 477
 Chapter 12 Computer Implementations and Exercises 483

Appendices 489

Appendix A: Sets and Basic Counting Principles 491
 Concepts and Notations for Sets 491
 Two Basic Counting Principles 495

Appendix B: Randomness and Probability 501
 Probability Terminology and Axioms 501
 Conditional Probability 507
 Conditioning and Bayes’ Formula 509
 Random Variables 511

Appendix C: Solutions to All Exercises for the Reader 515
 Chapter 1: An Overview of the Subject 515
 Chapter 2: Divisibility and Modular Arithmetic 517
 Chapter 3: The Evolution of Codemaking until the Computer Era 522
 Chapter 4: Matrices and the Hill Cryptosystem 526
 Chapter 5: The Evolution of Codebreaking until the Computer Era 530
 Chapter 6: Representation and Arithmetic of Integers in Different Bases 536
 Chapter 7: Block Cryptosystems and the Data Encryption Standard (DES) 540
 Chapter 8: Some Number Theory and Algorithms 545
 Chapter 9: Public Key Cryptography 550
 Chapter 10: Finite Fields in General, and $GF(2^8)$ in Particular 554
 Chapter 11: The Advanced Encryption Standard (AES) Protocol 560
 Chapter 12: Elliptic Curve Cryptography 563

Appendix D: Answers and Brief Solutions to Selected Odd-Numbered Exercises 569

- Chapter 1 569
- Chapter 2 572
- Chapter 3 581
- Chapter 4 587
- Chapter 5 592
- Chapter 6 595
- Chapter 7 599
- Chapter 8 601
- Chapter 9 604
- Chapter 10 608
- Chapter 11 609
- Chapter 12 611

Appendix E: Suggestions for Further Reading 615

- Synopsis 615
- History of Cryptography 615
- Mathematical Foundations 615
- Computer Implementations 616
- Elliptic Curves 616
- Additional Topics in Cryptography 616

References 619

Index of Corollaries, Lemmas, Propositions, and Theories 623

Index of Algorithms 625

Subject Index 627